VALUES AND VALUES STATEMENT EMERGED IN STUDENTS' PREFERENCES ON TEST ITEMS: A CASE STUDY FROM MATHEMATICAL INDUCTION

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ABSTRACT: Considering values from different aspects of the nature of mathematics and mathematics learning, this paper addresses the issues of students' values emerged in learning the concept of mathematical induction. Using three batteries of questionnaire survey and interviews, the primitive value of "uncertainty", "fun", and "certainty" and some derivative values will be discussed, in reference to the associated value statements in which such primitive values are contained. These students' carried values are then compared with the intended values of the teacher. As a result, the coincidence of the values in teacher designation of and students' selection from a group of 6 test items will be examined. Finally, a model of conceiving values, value statements, and identities from a syntactical and psychological point of view is proposed.

1. INTRODUCTION

Based on the results from a first year research report, pedagogical values were conceived as "mathematics teachers' principles for selecting and judging certain pedagogical identifications on the basis of whether they are of importance or worth to his or her classroom teaching of mathematics" (Chin and Lin, 2000, p.99). As a result, several salient values of an expert mathematics teacher were analyzed and reported. In the second year of the study, focus was moving to explore students' values that were emerged in and connected to T1's classroom teaching activities. Three core values, "**uncertainty**", "**fun**", and "**certainty**" obtained from an analysis of students' preferences on test items concerning concepts of mathematical induction will be reported in this paper.

In this section after briefing the aims of this three-year research study, aspects of conceiving teacher and student values are then addressed, followed by a discussion of three core values connecting to the nature of mathematics and mathematics learning.

1.1 Overview of the research project

The aims of this three-year research project on studying values in mathematics classroom were to explore the educational values that mathematics teachers have concerning mathematics and pedagogy, and its effects on their classroom teaching and student learning.

In the first year, the five core pedagogical values of an expert mathematics teacher (T1), who has taught in high school for 21 years, were described and reported (see Chin and Lin, 2000). In the following year of the study, focus was moving to describe the values of a group of six students from T1's class, emerged from selecting 2 out of the 6 test items concerning concepts of mathematical induction. Possible relationships of teacher and student values were also examined according to interviews of the six sample students. In the third year, to broaden our understandings of the nature and content of values, the study aimed at re-examining the stability of the methodologies used and extending the results found in the previous two stages of the research by studying two new participant teachers' and their students' pedagogical values.

1.2 Values statement as carrier of values

A values statement, as Taylor (2000, p.157) indicated, is a goal-directed description, indicating the values by which the school intends its practices to be guided, setting out the values the school intends to promote and which it intends to demonstrate through all aspects of its life. One example of a secondary teacher's values statement is "We want our school to be caring and Christian, disciplining, encouraging, happy." Such a statement then contains the values of **care**, **discipline**, **encouragement** and **happiness** that secondary schools are expected to addressed across the curriculum. Values, in light of this view, can be realistically and effectively existed in human talk and discussion, as being expressed in and entertained with statements of teacher or student's conceptions about mathematics and pedagogy; exist within the realm of human discourse (Haydon, 2000). Therefore, values in school typically in mathematics classroom, are public, objective, inter-personal, inter-subjective, and institutional (Aspin, 2000).

An example of secondary mathematics teachers' values statements have been reported in a recent article written by Chin and Lin (2000), in which the teacher evaluated the importance of his classroom teaching activity by the principle "Mathematics teaching is an activity to initiate desire, expectation, and enjoyment of knowledge." This statement then contains several educational values concerning learning and teaching of the school mathematics, referring to the values of student's **desirability**, **expectance**, and **enjoyment** in the process of learning mathematics. Several peripheral values statements were "Mathematics teaching is an activity to increase students' motivation and anticipation for learning", "Mathematics teaching seeks to teach students the nature of mathematical knowledge rather than mathematical forms", and "Mathematics teaching seeks to motivate students' interest and willingness to learn". These statements then seem to carry several derivative values, related to the previous three primitive values, such as the values of **motives** and **interests** for mathematics learning, and the values of **substance** and **forms** for the mathematical knowledge. The other four clusters of values statement and the relevant values carried are also given in the same paper (Chin and Lin, 2000, p.84).

We therefore conceive such a statement of values as the carrier of values, containing a teacher's principles for selecting and judging certain pedagogical identifications on the basis of whether they are of importance or worth to his or her classroom teaching of mathematics. In this case, values statement texturalizes teacher's educational values of mathematics and pedagogy into a concise sentence as the principle of instruction. As a carrier of values, it then contains a set of values that mathematics teacher values. This position of looking at values, from the realm of human discourse in terms of a syntactical and psychological analysis of language and words, will be further elaborated in the final section 4.2.

1.3 Values as teachers' and students' pedagogical identities

Values have been defined as “dually individual-social phenomena, the social identities concerning mathematics and pedagogy”, as “the principles or standards of teacher choices and judgements on the importance or worth of using certain pedagogical identifications in his or her classroom teaching of mathematics” (Chin and Lin, 2000). One of such principles for example, for the teacher to think about and act on classroom teaching from the aspect of mathematical knowledge, was “Mathematics is a useful and interesting subject”. This principle therefore relates to a teacher with an identity of seeing himself as a tutor of student learning committed to values of practicality, utility, and fun. As a result, his value system was construed as “the internal-external dialectic of identification”, as “the dialectical relationship between the varieties and complexities of individual pedagogical values”, or the process whereby all values are constituted (Jenkins, 1996, p.20). An example of such system can be found in Chin and Lin (2000, p.97), which configures a teacher’s value system from five different but related aspects referred to as ‘social’, ’educational’, ‘mathematical’, ’mathematics educational’, and ‘pedagogical’.

In the light of this definition, pedagogical values are seen as personal principles of thinking and practicing certain pedagogical identities with which the teachers identify themselves. Thus, we conceive the values of both teachers and students as their ‘Pedagogical Identities’ concerning mathematics and pedagogy developed through a dialectical relationship between the varieties and complexities of individual pedagogical identifications. They are the results of a process of an internal-external dialectic of identification. Students’ values, in this case, can be defined as “the principles or standards of student choices and judgements on the importance or worth of using certain pedagogical identifications in his or her classroom learning of mathematics”. For instance, one of such principles for students to think about and select on test items from the aspect of mathematics and mathematics learning, was “Learning mathematics is an interesting and thoughtful activity in which the process of individual thinking is of importance”. This principle relates to students with an identity of conceiving self as a meaning maker in the process of learning committed to values of “uncertainty” and “fun”, which will be discussed in section 3.1.

We consider both teacher and student to be a member of their social group to which they belong. The pedagogical values then reflect teacher and student’s principles of selection and judgement with regard to certain identities concerning mathematics and pedagogy, which are shared among group members (teachers or students). These identities describe not only their personal characteristics of mathematics teaching and learning, but also the shared characters of the teacher and student group. These shared identities reflect much of the specific features teachers and students have within their context of schooling. The identities thus act as a set of consistent and coherent schemas showing both the conceptual and behavioral aspects of teacher in teaching mathematics and student in learning mathematics.

1.4 Values in mathematics and mathematics learning

From student’s aspect, typically that of mathematics and mathematics learning, three sets of values relevant to the substance of the teaching and learning of mathematical induction will be brought out for discussion. They have been discussed in the book entitled The Mathematical Experience by Davis and Hersh (1981). One set of values derives from the nature of mathematics as the science of "**infinite**" and as the science of patterns related to "**generalization/generality**". Another set of values relates to the nature of mathematics learning, such as "**fun**" and "**free thought**". The third set of values is included in a view of conceptualising mathematics as the science of formal systems, entertaining the values such as "**forms**", "**proofs**", and "**mathematical language**".

A typical value judgement on the nature of mathematical knowledge, that Davis and Hersh indicate, is "Significant mathematics is thought to emerge when the universe of its discourse is enlarged so as to embrace the infinite"(p.152, ibid). This sentence might be re-written as "Mathematics becomes significant for it embraces **infinite**". This values statement then carries the value of infinite, playing a crucial role in the extension of mathematical knowledge to the concept of infinities. The value of infinite includes several generic concepts such as "endless", "eternity", "immortality", "mystery", and "self-renewability". These genres create the essence of the concept and development of mathematical induction, bridging the gap between the finite concept of scientific induction (finite induction) and the infinite nature of mathematical induction (infinite induction). It is this value (i.e. infinite) that makes mathematical induction valuable and worth of learning.

The dual role that specialization and generalization play in the process of the invention of mathematical knowledge is well known for mathematicians and mathematics educational researchers. An example of the evaluation on the role of generalization and specialization shown in the values statement "One benefit of **generalization** is a consolidation of information" (p.135, ibid), is that generality plays an active role in creation of the mathematical knowledge. A good example is a seven-stage generalization process of examining the development of "The Chinese Remainder Theorem" (p.187-195, ibid). The value of generalization in this case is to complete the development of the theorem from its chaos to orders. It also demonstrates the derivative value of **beauty** in the process of creating such mathematical knowledge. The concept of mathematical induction reveals also its power of generalization by subsuming the finite concept in an infinite domain. It is this second value that empowers the concept to a general and abstract case mathematically.

In arguing the question of whether mathematics is invented or discovered, Williams (the Chairman of Mathematics Department) professes with "The thing that is important is that doing math is fun. That's what I try to put across to the kids" (p.272, ibid). This then infers a values statement as "Learning mathematics is interesting and doing it is a pleasant thing" which carries the value of "**fun**", associated with a value "**pleasure**" derived from the pursuit of fun. As a teacher, Williams concludes with "What I try to do is to tell math to kids on the basis that it's fun", a sentence that carries the value of fun.

In discussion of the non-analytic aspects of mathematical knowledge within context of the school mathematics, referred to as unconscious and analog mathematics, they indicate that the unconscious mathematics creates one aspect of mathematization, in which analog and analytic mathematics share. The analog mathematics is related to the idea of intuition and feeling, in which "sense making" and "free thought" are the two important elements are relevant to the learning of mathematical induction. These are also two values that students should value in the process of learning mathematics. On the other hand, the analytic and conscious aspects conceive mathematics from the other way round as a combination of forms, rules, and proofs. This then is related well to the concept of mathematical induction as being formalized in schools and classroom teaching as a step-by-step mechanical process of proofs.

Several values statements concerning the nature of mathematical knowledge are “Every text on mathematical logic explains the rules of syntax for this language” (p.136, ibid), “Mathematics is uniquely characterized by something known as proof” (p.147, ibid), and “A formalized text is a string of symbols”(p.139, ibid). These statements then carry the value of “**forms**”, “**proofs**”, and “**mathematical language**”. These values also include several relevant concepts such as “if-then logic”, “symbols”, “vocabulary”, and “truth”. These genres create the essence of the concept and development of mathematical induction, subsuming the finite aspect of mathematics into its infinite domain. These values therefore make mathematical induction valuable and worthy of learning.

The triad that language, proofs, and forms play in the creation of all aspects of mathematics is well known for mathematicians and mathematics educational researchers. An excellent example is given and characterized by a two-step mathematical processing of the concept of mathematical induction. This reveals also the power of formalization that mathematics creates in the learning of mathematical knowledge. It is these values that empower the concept to a highly abstract and powerful level.

2. RESEARCH METHOD

2.1 A framework

Questionnaire surveys and interviews were used in investigating a student class of T1 and six student samples. Data were collected at two stages, one was during October 1998, after topic of mathematical induction was taught; the other was during April 1999, after topic of trigonometric functions was taught. Two formats of student questionnaire were designed and used in these two stages. First questionnaire was designed to uncover all students’ preferences on selecting two of the six problems relating to the topic taught, and also to collect students’ reasons of doing so. First part of this questionnaire consists of six open-ended problems designed by T1 based on his core values of mathematics and pedagogy, and the second part has several items for students to express their degree of agreement on each statement, concerning the reasons of doing and not doing so.

The second questionnaire consisted of 20 questions, using a five-point Likert Scale to show varied degrees of agreement on each item for all students to choose. It is then asking students to express their positions of agreement on each question according to two different contexts, test scores taken for grading or not to be taken. A third and follow-up questionnaire, consisting of 86 items, was used in interviews to 6 selected students, as an aid for the researcher to catch the reasons of these students’ preferences of choices and judgements on test items.

Above three questionnaires were piloted and revised according to the responses of another student class in same school with same grade.

2.2 Instruments

In this section, we will describe some items from each questionnaire as exemplars. Three out of the 6 questions used in first part of the first questionnaire were:

1. Please give a realistic example in which the underlying concept is relevant to mathematical induction.
3. Suppose that the concept of human is well defined, and the life in the earth has gone through about 4 billions year. Prove that there is a human being whose mother is not a human.
5. Prove that $3^n \geq n^3 \ \forall n \in \mathrm{N}$

The question no.5 (called 'Formality') above is popular for tests in this topic, and it is also a question with which the students are familiar. This item, as an exercise, clearly was designed for the purpose of testing student understanding the technical procedure and forms of the concept of mathematical induction. It was used to examine student understanding from a "conscious or analytical aspect" (Davis and Hersh, 1981, p.301-316) of mathematical induction. However, the question no.3 (called 'Genesis'), as a realistic problem, is something very different. It was used to examine student understanding and the ability of application from a "unconscious or analog aspect" (Davis and Hersh, 1981, p.301-316) of mathematical induction. The question no.1 (called 'Exemplar') was designed from "a phenomenological aspect of mathematical concept" (Freudenthal, 1983), asking students to find out some examples from the real word to which that the concept of mathematical induction might be applied.

Second part of the first questionnaire was asking respondents to choose or write down their principles or reasons of selecting some questions rather than others. One out of the 5 items was:

> Please click the appropriate statement shown as follows or write down your personal descriptions, concerning the reasons of choosing the two questions.
> I chose __ (question number) because:
> (1) the question is familiar to me
> (2) the question is easier for me
> (3) the question let me understand the meanings of mathematical induction
> (4) it is interesting for me
> (5) it is useful in practice
> (6) it is a question that might be used in tests
> (7) it reflects the ideas that T1 has stressed
> (8) other reasons ___________________

The second questionnaire consisted of 20 items, took a five-point Likert Scale format (score 1 to 5) by asking students to express degree of agreement on each statement, according to two different contexts, test scores taken for grading or not to be taken. One example is:

I select the questions that are more interesting for me.

Finally, a third battery of questions consisted of 86 questions concerning to varied aspects of mathematics and mathematics teaching. For example, different views of mathematics, attitudes of mathematics, views about mathematics learning and assessment, ways of instruction, etc. Three examples were:

04. Students can understand mathematical concept on their own.
09. The goal of mathematics teaching is to increase students' test scores.
10. Mathematics is mainly to do with calculation.

These questions were employed in the interview for 6 sample students, to uncover the basic principles that they used in choosing and answering items of the previous two questionnaires.

2.3 Values implicit in the questions

Several values, that 'Genesis' and 'Formality' carry in the first questionnaire discussed above, T1 used to examine his student class will be brought out in this section. The value of practicality or utility, as Davis and Hersh mention (1981, p.79-89), is related to both of the concrete-applicable and abstract-formal levels of thinking and reasoning. 'Genesis' seems to carry such values as "**practicality/utility**" and "**fun**" (Davis and Hersh, 1981), and "**spoken language**" (Pimm, 1987), in which the ideas of usefulness and talk are central. However, 'Formality' entails the value of mathematical "**forms**" and "**proofs**", and "**mathematical language**" (Davis and Hersh, 1981; Pimm, 1987), in which rules, symbols, and calculation are of importance. As far as the concept of mathematical induction is concerned, its basic values as discussed in section 1.4 are "**infinite**" and "**generalization/generality**"

In this case, the implicit values that these questions entail are then tailored into a context of assessment for students to choose. It was supposed that in the process of choosing among alternatives that students' values would become explicitly.

2.4 Interviews

6 sample students were selected according to the following three criteria: mathematical performance in the test, willingness to talk, and representative of the student group. "Dialogue interviews" (Chin and Lin, 2000), including reflective and introspective discussion, and recursive probing procedures, was employed in exploring those students' values in terms of the three instruments above, in which the students played an active role in the conversation while the researcher acted more or less as an inquirer. A set of probes was also used in such dialogues for instance, "What were the reasons for you to choose these 2 rather than the rest 4 questions to solve? What the most salient impressions did you get during the section of learning mathematical induction? What kinds of messages do you think that T1 was trying to pass on to you through such section of teaching? How did you feel about that?"

3. RESULTS

Two value statements were found to be salient in the interviews. One was

“**Learning mathematics is an interesting and thoughtful activity in which the process of individual thinking is of importance**”, derived from the students who chose the two ‘Genesis’ like problems (no.3 and no.6). The other one was “**Learning mathematics is an activity of solving given questions in which calculation and definite answer are two crucial elements**” observed from the students who selected the two ‘Formality’ like problems (no.5 and no.2). Three implicit values, contained in these two value statements, will be reported here: **uncertainty**, **fun**, and **certainty**.

We will discuss the implicit values that a group of 6 students shared, according to their responses in the interviews and questionnaire survey, typically using the case of S1, S2, and S4. S1 and S2 chose the only two conceptual problems, while S4 selected the only two technical problems.

3.1 The value of uncertainty and fun

As table 1 shows, a value statement related to the high mean scores found across the 5 items under the condition of ‘no examination required’, seems to be “Learning mathematics is an interesting and thoughtful activity in which the process of individual thinking is of importance”. This suggests that “Mathematics is a realistic and open subject to learn, and should be learned by students themselves“.

Table 1: Student responses on 5 selected items of the second questionnaire

Items	Mean Score	
	No Exam.	Under Exam.
01. I choose questions that are non-familiar for me.	4	2.4
02. I choose questions that are interested for me.	4.4	3.3
03. I choose questions that are brand new for me.	4.3	3
07. I use my own way to solve the questions I chose.	4.3	3.2
09. I choose questions that encourage thinking freely.	3.7	1.9
Total Average	4.14	2.76

When authors asked ‘Why did you choose question no.3 and no.6 (‘Genesis’ like problems) rather than the other 4 questions?’ Two students’ (S1 and S2) responses at several different stages of interviews are summarized as follows:

> S1: Because I felt that these two questions were intended to ask me to *elaborate carefully*, and *the words* used in portraying the problem’s context do make sense (means easy to follow) to me.
>
> S1: These are questions that I *have not seen before*. They are *new* for me. But, I am quite *familiar* with the other two questions (‘Genesis’ like problems) that I am not so interested in solving them. They are so *boring* because I have already known the answer of the questions and the procedure of solving them.
>
> S1: They seem in some way connected to *the definition* of mathematical induction, and they are *interesting* too.
>
> S1: The questions raise my *curiosity* to solve them and I like to find out myself. I really like to know whether if I can solve *a totally new problem* like this *on my own*. These were the reasons why I chose them.
>
> S1: There is *no pleasure* for me to face a mathematical question which has

no practical use or *not realistic*. I don't know what could be of interest if you have already known the answer or method of solving the question?

S1: I like to solve it by using the way that is developed through my *own construction*. There are lots of *pleasant* during the processes of solving such question.

S2: I am not so interested in the questions such as no.2 and no. 5 ('Formality' like problems), because *you can find them everywhere* in books, or in lessons, or in tests. They are so popular for me as a senior high school student.

S2: I should say that they are the questions that might come out in all kind of the school tests, but you *would never find such problems as no.3 or no.6 ('Genesis' like problems) in any tests*. I mean any tests including in and out school tests.

S2: I would have *got bored* if a mathematical question were solved easily according to certain familiar steps. It is *no fun at all* for doing or answering a question like that.

S2: I supposed that you were trying to understand or collect my principles of choices or decisions on these 6 problems? If it is what you want me to do then I should let you know my real decision, otherwise, I could cheat you by choosing no.2 and no.5 ('Formality' like problems) isn't it?

S2: I am now telling you that I hope all mathematical are like this, encouraging me to *think freely*, fulfilling my *curiosity*, and being full of *pleasure* in the process of solving them. Moreover, the meanings of *the language* used in such questions were easy to be understood.

What is critical for these two students to choose 'Genesis', was that the problems should be setting out in the context providing "unknown", "thinking free", and "naïve spoken language" entailed with sort of "curiosity" and "interest". In other words, a principle that they considered in the process of choosing these 6 items was "**Learning mathematics is an interesting and thoughtful activity in which the process of individual thinking is of importance**". This values statement as a carrier of values, seems to suggest that the value of "**uncertainty** " and "**fun**" are primitive and of paramount importance in the process of evaluating what mathematical problems should be solved in the test. In the light of this, several derivative values were "unknown", "free thought", "spoken language", "curiosity", and "interest".

3.2 The value of certainty

As shown in table 2, a value statement related to the high mean scores found across the 4 items under the condition of 'examination required', is thought to be "Learning mathematics is an activity of solving given questions in which calculation and definite answer are two crucial elements". This seems to mean that "Mathematics is a formal and given subject to learn, and should be learned by following the logic of rules and forms".

Table 2: Student responses on 4 selected items of the second questionnaire

Items	Mean Score

	No Exam.	Under Exam.
04. I choose questions that are familiar for me.	2.9	4.3
05. I choose questions that are easy to solve for me.	3.3	4.6
08. I use formulae to solve the questions I chose.	3.3	4.1
10. I choose questions that can easily be calculated.	2.9	4.3
Total Average	3.1	4.33

When authors asked 'Why did you choose question no.2 and no.5 ('Formality' like problems) rather than the other 4 questions?' S4's responses at several different stages of interviews, are summarized as follows:

> S4: I liked the questions ('Formality' like problems) because they were asking me to *calculate*. Through the procedure of calculation that I know before, I *feel more confident* about the solution. Besides, this procedure would lead me to *a definite answer* that the questions have set up. Then I could solve them successfully.
>
> S4: They are also the problem with which I am *familiar*, and I don't like questions that don't have *definite answer*.
>
> S4: There are too many words in question no.3 and no. 6 ('Genesis' like problems). They do not make any sense for me, since they seem to be *no approachable answers* as far as I know. This isn't right based on my personal experiences with solving mathematical problems.
>
> S4: They were interesting for me because the questions were *familiar* to me. These two questions ('Formality' like problems) are easy to *get the right answers* through *calculation*, and most importantly, they are easy to be solved.

What important for S4 to choose 'Formality' were that such problems should be setting out in the context providing "definite answer", "familiarity", "calculation", and "mathematical language" entailed with a special kind of "interest". But, the meaning of interest for S4 was quite different from that of S1 and S2. Such interest was based on certainty of getting the right answer and familiarity of the problem.

In other words, a principle that they considered in the process of selecting these 6 items was "**Learning mathematics is an activity of solving given questions in which calculation and definite answer are two crucial elements**". This values statement as a carrier of values, in this case, suggests that the value of "**certainty** " is primitive and of paramount importance in the process of evaluating what mathematical problems should be taken in the test. In this case, several derivative values seem to be "known", "familiarity", "mathematical language", and "calculation".

4. DISCUSSIONS

We will try to analyze the relationship between teacher and student's values, followed by a discussion of the relevance of values, values statement, and identities.

4.1 The values that teacher values and the values that student values

The values carried in the test and the values that students took in the questionnaire surveys and interviews can be re-organized into the following table 3. The corresponding values between the teacher and students as written in 'bold' mean their values were straightly correlated while others stand for on direct correlation or missing according to our collected data.

Table 3: A comparison of values from the teacher and student's aspect

Teacher's Intended Values	Items	Students' Carried Values
Practicality/Utility ***Fun*** ***Spoken Language*** Infinite Generality/Generalization	'Genesis"	Uncertainty ***Fun*** (unknown, free thought, ***spoken language***, curiosity, interest) ? ?
Forms Proofs ***Mathematical Language*** Infinite, Generality/Generalization	'Formality'	Certainty (known, familiarity, ***mathematical language***, calculation) ? ?

The values that T1 value and the values that his student values, as being examined through the selection of 6 test problems, show varied degrees of coincidence. While solving 'Genesis' (no.3 and no.6), students might find a sort of usefulness, curiosity, and interest in the solving process that create "fun". The language and the rich context set up in such a problem may as well get students a feeling that mathematical knowledge in such a situation is easier to get access. This then creates the values of "spoken language" in the student site, which is coincident with the teacher's. Moreover, in solving such a question, students naturally entailing a feeling of unknown and autonomy in thinking which then create a new value "uncertainty". Some relevant protocols found in the interviews are summarized as follows:

S1: His way of instruction fit to that of mine. I think that mathematics should be learned through this way, using *a general language* in describing the context and getting me a feeling that *the answer of the question is not quite sure*. I like this feeling of *unknown*. It also created my *curiosity* to solve the problem.

S1: I am more interested in attending such a class, since in such a class I can always *think freely*.

S1: He showed us a way of *thinking and constructing of our own* in mathematics lesson. This was extremely different from the traditional classes that were well known and accepted in this topic. It did create my *eagerness* to find out the solution by myself.

On the other hand, when students tried to solve 'Formality' (no.2 and no.5), they may get a feeling that the questions are full of mathematical forms, rules, and symbols in the process of finding solution. This then may encourage students to create the value of "certainty" and "mathematical language". In the process of solving such question, student might eventually feel that the answer must be already known and the question is popular and familiar for them. The only thing they can do is to follow a step-by-step procedure of calculation to get the answer rightly. Some of the protocols found in the interviews are summarized as follows:

> S4: I like a question that has a *definite answer*, because it was accompanying *certainty* with me in the solving process. I like to solve mathematical problems in such a mood.
>
> S4: Questions like these two enable me to solve them under a climate with which I am *familiar*. In this case, I can follow the procedure of *calculation* that I have *known* to find out the answer.
>
> S4: I don't think it is a good idea if a mathematical question let people feel that the question may have no definite answer.

However, the mathematical values of the concept concerning "infinite" and "generality" seemed to be missed on student site. This may due to the research design or to the test format, because students are not recognized the values that such problems entail. Therefore, it seems to us that students value what they think to be valued according to their values about mathematics and mathematics learning, and teachers on the other hand, can only prepare as far as they could the values varying across the topic for students to choose and identify.

4.2 Values, values statement, and pedagogical identities

Value statement, as carrier of implicit and explicit values, is a concise and crucial sentence that teacher and student use in portraying their positions or principles of evaluation on teaching and learning incidents. A text like this has its syntax or structure of wordings in which certain values may be entailed implicitly or explicitly. Those critical words that are subsumed in values sentence might represent certain values that have direct or indirect meanings relating to such words. A re-phrased values statement such as "Learning mathematics is an interesting and thoughtful activity in which the process of individual thinking is of importance**"**, suggests the person who produces the statement identify *indirectly* the values of "uncertainty" and "fun" as being discussed in section 3.1. An original protocol, on the other hand, relates to the values *directly*, as the following three relevant sentences, connected to the above core sentence, that S1 gave in the interview:

> The questions raise my *curiosity* to solve them and I like to find out myself (fun). I really like to know whether if I can solve *a totally new problem* like this *on my own* (uncertainty). These were the reasons why I chose them.

A value statement, in the light of this, can be derived directly from the original sentence that a value carrier produces, or being indirectly inferred on the basis of the relevant interview protocols. Values are in this sense represented and embodied through words or a combination of words such as "uncertainty", "fun", and "certainty" discussed above. A set of values represented by words has brought out for examination and discussion in our Australia colleagues' report (see Clarkson, Bishop, FitzSimon, and Seah, 2000, and FitzSimon, Seah, Bishop, and Clarkson, 2000, this volume). For instance, the value of "clarity", "enjoyment", and "conjecturing" about mathematics education and the value of "logical thinking" and "creativity" that the teachers portrayed and preferred. Other examples of representing values in words are given in The Agreed Minimum Values Framework documented and developed by the NPDP Value Review Project in Australia, for example “knowledge”, “equity”, and “quest for truth” (Caple, 2000, p.210).

Any ones, who agree or identify oneself with such direct or indirect value statement, are conceived as carrying the underlying fundamental elements of the statement, that is, the values that the sentence embodies. This is to conceive and analyze values from human discourse in terms of a syntactical and psychological analysis of language and words. It is this process of pedagogical identification that reflects a person’s preferences to the foundations of mathematics and pedagogy. It is such a foundation concerning mathematics and mathematics teaching that connects to the pedagogical identities of the teacher and students. In this sense, values are integral to self-identity and that teacher and student strive to be authentic (Rokeach, 1973, 1979). Such view of seeing values as being driven by a teacher’s identity are also proposed by our Australia colleagues, as they put it (Bishop, Clarkson, FiztSimons, and Seah, 2000, this volume):

> A number of teachers who believe that mathematics learning has value for their students, may have never considered the particular values they are imparting. The values taught, whether explicitly or more like implicitly, seems to depend heavily on one’s *personal set of values as **a person** and as **a teacher***.

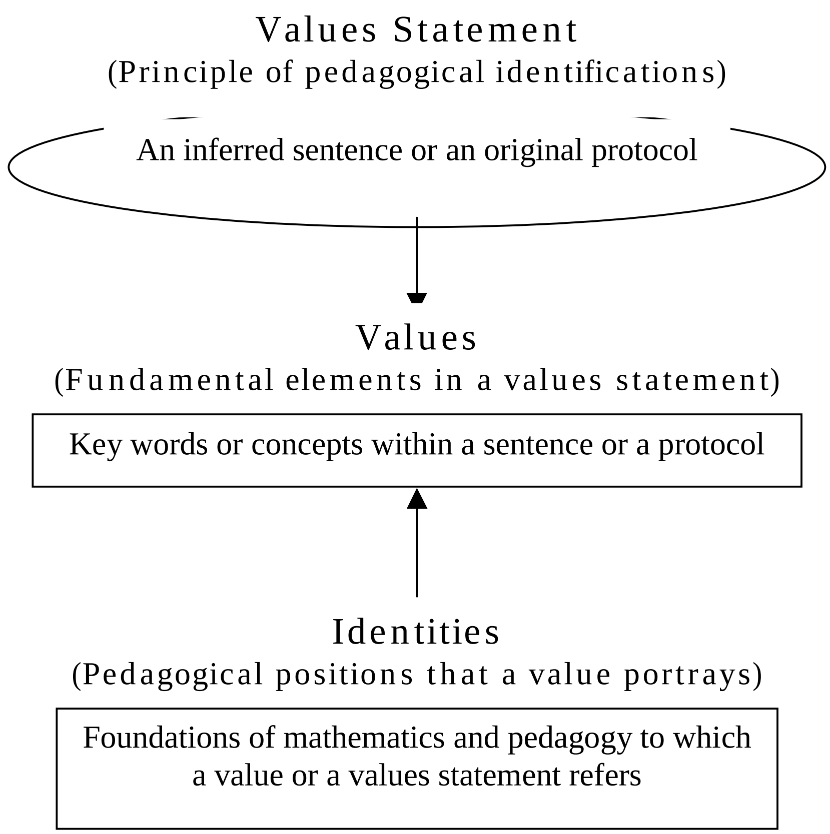

Figure 1: A portrayal of the correlations between values, values statement, and identities

In addition, Halstead (1996, p.5), in searching for a proper definition of values, also comes to a conclusion referring the term values to individual identity:

> The term values is used to refer to principles, fundamental convictions, ideals, standards or life stances which act as general guides to behaviours or as points of reference in decision-making or the evaluation of beliefs or action and *which are closely connected to* ***personal integrity*** *and* ***personal identity***.

Thus, in the assessment of learning outcomes of mathematical induction, T1 and his students showed us "How teacher and students identify themselves with certain values through the process of designing and solving the 6 value-laden mathematical problems". Although, in doing so inevitably involves the researchers' own beliefs of interpreting values carried by the relevant values statements, and also the difficulties in finding an appropriately shared language between teachers and the researchers, as we and our colleagues have learned (Chin and Lin, 1999; Clarkson et. al., 2000).

In summary, a relationship between the pedagogical values that mathematics teacher and student value, the values statements in which such values are carried, and the identities that the value carriers identify, is represented in Figure 1. The figure is used to describe the mental process of pedagogical identifications concerning mathematics and pedagogy that an expert mathematics teacher and his student class mediate, in the context of teaching and learning such topics of the school mathematics as mathematical induction.

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