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Values in mathematics textbooks: A view through two Australasian regions

**Paper presented at the 81st Annual Meeting of the
American Educational Research Association**

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This paper outlines a preliminary investigation into the kinds of mathematical and mathematics educational values conveyed in representative lower secondary mathematics textbooks in Singapore and Victoria, Australia. Mathematical and mathematics educational values are viewed as being influenced by --- and acting on --- different socio-cultural levels of values. Data compiled from the content analysis exercise reveal an unbalanced portrayal for each of the eight selected pairs of complementary values. The relationship between some of the commonly-adopted value signals and the nature of mathematical genre is explored. Differences in the way values are portrayed in the two culturally different regions are also discussed.

“Education socializes people to ... official values” (Neuman, 1997, p. 403). “Education, the process of continued socialization, is aimed at specific socio-politico-economic goals far beyond acquisition of the ‘Three Rs’ “ (Eckermann, 1994, p. 39), specifically to inculcate the individual and the citizen in each pupil. Such values are often taught through school subjects and through interpersonal interactions in school, explicitly or implicitly. It is probably evident that the value of teamwork is often portrayed to --- and inculcated in --- pupils in physical education classes, that of national pride through history lessons, and environmental care through the context of a particular language class. The perception that mathematics is the same objective knowledge everywhere in the world, and the fact that mathematical knowledge is essentially context-free (despite the inclusion of context in mathematics pedagogy) both lead to the commonly-held view that mathematics is culture-and value-free.

Relatively recent studies in ethnomathematics (see, for examples, Ascher & Ascher, 1994; Bishop, 1994; Borba, 1997; D'Ambrosio, 1985; D'Ambrosio, 1995; Gerdes, 1994; Gerdes, 1996; Knijnik, 1993; Pinxten, 1994) have shown that different mathematical practices are conceptualized and practised in different cultures, some of which can be very efficient (Borba, 1997). The development of multicultural mathematics education (see, for examples, Banks, 1997; Erickson, 1997; Fasheh, 1982; Joseph, 1993) is also an acknowledgement that pupils from different socio-cultural backgrounds bring with them different perspectives and knowledge of mathematical practices.

Thus, "every people, every culture and every subculture, including every social group ... and every individual, constructs and develops its own, in a certain way *particular*, mathematics" (Gerdes, 1998, p. 47). In fact, "[one] of the greatest ironies ... is that several different cultures and societies have contributed to the development of what is called western mathematics: the Egyptians, the Chinese, the Indians, the Arabs, the Greeks, as well as the Western Europeans" (Bishop, 1990, p. 61). In other words, "school mathematics is mathematics as it is conceptualised, represented, structured, and sequenced to share with the next generation through the formal schooling experience" (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997, p. 4).

In this context, this paper reports the results of a preliminary investigation into the kinds of values portrayed in representative lower secondary mathematics textbooks in two Australasian regions, i.e. Singapore and Victoria, Australia.

Significance of study

Although cognitive objectives have affective components, and vice versa (Krathwohl, Bloom, & Masia, 1964), cognitive educational objectives appear to have received a greater emphasis in curriculum statements, in textbooks, and in teachers' conscious teaching practices. A possible reason is that since we are better at handling the cognitive domain, we often focus on cognitive objectives as a means to achieving affective goals. Another related reason may be that the extent of attainment of cognitive reasons is easier to measure. A failure to place a greater emphasis directly on the affective aspect of education, however, can be dangerous especially in an age of lifelong learning and continual retraining, and when increasingly high premium is being placed on the quality of citizens. As Hill (1991) notes,

apart from anything else they might learn, students get the message that, in the things which it includes and excludes, the curriculum mirrors the priorities which the community sets on things Values education goes on, therefore, even when we are not consciously planning for it. But when its effect is not acknowledged or controlled, the result is often that wrong values for life are propagated by default. (p.3)

Moreover, affective objectives do facilitate the achievement of cognitive goals (Bloom, 1976; Eckermann, 1994; Turney, Eltis, Towler, & Wright, 1986). For example, teachers and textbook writers may aim to increase pupil interest and motivation in mathematics by presenting content in a fun, easy-to-understand way. The confidence possessed by a pupil is likely to provide him/her with a special energy when he/she approaches a problem, an internal boost which will increase his/her productivity and creativity. After all, values involve a greater level of cognitive involvement compared to other less internalized affective qualities like beliefs and attitudes (Krathwohl et al., 1964).

Mathematics curriculum statements in Singapore and Victoria do refer to the communication of different values to pupils. In Singapore, for example, the lower secondary mathematics syllabus expresses an aim that pupils are to be able to "recognise the relationships between topics [and to] become aware of the application of mathematics in other subjects" (Singapore Ministry of Education, 1990, p. 5). Thus, there is a promotion of relational understanding (Skemp, 1979) among topics within mathematics, and between mathematics and other subjects. The Victoria mathematics 'Curriculum and Standards Framework II' (CSF) states that students "should learn simple

ways of ... justifying their work” (Board of Studies, 1999, p. 214), thereby communicating the message that working mathematically involves a level of being open to peer scrutiny, being accountable, and acting transparently.

However, “from a single set of curriculum guidelines an infinite number of textbooks could be built, each with its own interpretation of the intent of the guidelines” (Venezky, 1992). In Australia, Shield’s (1998) analysis of local mathematics textbooks shows that they “do not convey the intent of recent reports and syllabuses, even though they were written in response to these documents” (p. 521).

From a wider perspective, not all values may be relevant, suitable or desirable in different societies. For example, in the Confucian-heritage culture of Taiwan where the mathematics curriculum is very much influenced by American educational ideas, questions are being asked about the possible consequences of unquestioned acceptance of values underpinning these foreign ethos (Chin & Lin, 1999).

However, mathematics has not enjoyed as much academic/research attention in affective issues as some other subjects, such as the languages, literature studies, history, physical education (e.g. Aplin & Saunders, 1996; Lee & Cockman, 1995; Murray, 1977), and the sciences (e.g. Allchin, 1999; Proctor, 1991; Tan, 1997). It may be that these other subjects deal with aspects of life experiences more directly and more explicitly, so that values can be easily associated and/or discussed with them. Mathematics, on the other hand, often deals with abstract entities and ideas, and with how these are applied to real-life situations (and indeed to many of the other school subjects mentioned above!).

Furthermore, research in the affective component of school mathematics has mainly focussed on gender, and attitudes and beliefs (e.g. Anderson, 1998; Bleicher, Cooper, Nisbet, & Warren, 1995; Bobis & Cusworth, 1995; Callingham, Watson, Collis, & Moritz, 1995; Carroll, 1995; Galbraith & Haines, 1998; Gervasoni, 1995; Kaleva, 1998; Perry & Howard, 1999; Perry, Howard, & Tracey, 1999; Philippou & Christou, 1998; Raymond, 1997; Thompson, 1984; Tirta Gondoseputro, 1999; Wang, 1999).

In fact, educational research in values has been both relatively recent and scarce. This is partly due to a rather fuzzy understanding of, agreement to, and distinction among the various affective variables such as attitudes, beliefs and values (Krathwohl et al., 1964; McLeod, 1992). Also, related terminology have been used interchangeably, as in religious faith/beliefs/values, leading many to conclude wrongly that the terms are synonymous. The word ‘value’ itself has several different usage in the language too, such as in the ‘value’ of an unknown in an equation, the ‘value’ of listening to a speech, and the (moral) ‘value’ of an individual. All these have contributed to some uncertainties with peers and teachers we have encountered in a current research project (Clarkson, Bishop, FitzSimons, & Seah, 2000, in press). Another contributing factor is that reliability of affective studies has generally been questionable in the academic field (Southwell, 1995).

Thus, the relative lack of research interest in values in mathematics education does not mean either that the entire subject of values is irrelevant to optimizing mathematics education in schools, or that the current state of knowledge about values education is sufficient enough to facilitate effective teaching and learning. As Southwell (1995) notes, the failure of recent and current educational initiatives to show any tangible

improvements has fuelled the current renewed interest in affective factors in mathematics education.

Values

What exactly, then, are values in general and in the context of the mathematics classroom? Kluckhohn (1962) defines a value as "a conception, explicit or implicit, distinctive of an individual or characteristic of a group, of the desirable which influences the selection from available modes, means, and ends of action". There is thus an ownership of values by the individual and by the community to which the individual belongs. In the same way, Raths, Harmin and Simon (1987) refer to values as "general guides to behavior" (p. 198) arising from one's experience in --- and relationships with --- the world. However, Swadener and Soedjadi (1988) appear to avoid situating values in any context, merely defining a value as "an idea or concept about the worth of something" (p. 197).

Three definitions in the early- and mid-1990s appear to situate values either as a personal characteristic or as a community-defined quality. Considering the former first, McConatha and Schnell (1995) see values as

primary constructs which affect an individual's interpretative schema and his or her sense of self, thereby exerting a direct or indirect influence on attitudes, beliefs, feelings, and the perception of the social and political world. In other words, values provide abstract frames of references for perceiving and organizing experience and for choosing among alternative courses of action. (p. 80)

Similarly, Hill (1991) refers to values as "those beliefs held by individuals to which they attach special priority or worth, and by which they tend to order their lives" (p. 4). He identifies three elements in the conception, i.e. cognitive, affective and volitional, stressing the importance to distinguish the volitional aspect from the other two because it needs not follow from knowing and feeling, and because of a possible infringement of personal rights to demand conformity to certain actions.

On the other hand, Nixon's (1995) definition is clearly community-referenced:

Values are important not because they provide logical explanations, but because they are asserted and require assent. They affect action by satisfying our sense of what feels right or awakening our sense of what is morally offensive. The affective nature of values --- the way they cling to feelings and associations --- accounts for their resilience and for the continuing influence they exert across generations. Values take us, as individuals and groups, back to our roots for the purpose of reclaiming what is morally alive in our communal pasts; they trace old loyalties but point also to new possibilities for realising our own moral agency and for supporting that of others. They can, and do, lead to the exclusivity of tribal and ethnic nationalisms, but they are also expressed in innumerable acts of altruism and self-sacrifice. (p. 220)

Similarly, Tan's (1997) definition in the late-1990s signals a return to Kluckhohn's (1962) opinion that values reflect an interaction between the individual and the community:

Values thus appear to be a form of conceptual and emotional goggles, a means to allow us to evaluate/see the worth of something, some action or goal. They tend to have a moral aspect involved and are concerned with human actions or behaviour and

to do with good and bad. They are part of the culture of a community, and help to guide the actions of its members toward each other and the community as a whole and toward other communities that may have different values. In some communities, the values extend to define the community or its members' relationship with nature, both living and nonliving. Values form the basis of a person's choice of action and attitudes toward others and toward the world at large. (p. 559)

Specific to mathematics learning and teaching, Bishop's (1996) definition of values also makes a comparison with associated forms of knowledge:

Values in mathematics education are the deep affective qualities which education aims to foster through the school subject of mathematics. They appear to survive longer in people's memories than does conceptual and procedural knowledge, which unless it is regularly used tends to fade. (p. 19)

Inherent in this definition is the influence educational institutions and systems --- and indirectly the socio-cultural and political entities as well--- exert in the formation of value systems in pupils. Also, there is the implicit reference to an active intention to teach values in schools, a point acknowledged at the beginning of this paper. There is also a subconscious aspect to values education --- such as in teacher portrayal of personally-held values through his/her interactions with pupils in the (mathematics) class --- of which those involved may not be acutely aware of at the moment of occurrence.

Theoretical framework

The conduct of this preliminary study has been informed by a theoretical research framework made up of the following three aspects:

- development of personal values,
- values in the mathematics classroom,
- mathematics textbooks as a source and medium of values portrayal.

Development of personal values

In their second taxonomy of educational objectives, which focuses on the affective domain, Krathwohl, Bloom and Masia (1964) conceive affective objectives as being located along a multidimensional internalization continuum (Chap. 3). These objectives are arbitrarily organized into the following five stages: (1) receiving (attending), (2) responding, (3) valuing, (4) organization, (5) characterization by value or value complex. Each of these five stages are subdivided into sub-stages.

Progressing from the receiving (attending) stage to the valuing stage, there is a greater level of internal control over the ownership of affective variables. Accompanying this greater individual control is the increasing complexity and abstraction of the affective variables.

Raths, Harmin and Simon (1987) introduce the idea of value indicators, which include attitudes, beliefs and interests. According to them, value indicators undergo through valuing processes to become values. The satisfaction of all seven criteria must take place for a successful valuing process:

- choosing --- freely
- from alternatives
- after thoughtful consideration of the consequences of each element

- prizing --- cherishing
- affirming to others
- acting --- with the choice
- repeatedly, in some pattern of life.

The relevance of some of these seven criteria are certainly debatable (see Seah, 1999; Stewart, 1987). For example, certain life experiences may create such deep impressions and strong impact on the individual that they shape or alter his/her outlook and attitudes to life, and indeed personal values forever, so that the criteria of values being the outcome of choosing after thoughtful consideration of the relevant consequences may not apply. In terms of Krathwohl et al's (1964) 'Taxonomy of Educational Objectives (affective domain)', this valuing process corresponds to the mid-level of the Taxonomy, i.e. the valuing level. It is at this and the two higher levels of the Taxonomy too that cognitive involvement of the individual becomes significant with the corresponding decrease in affective response.

Beyond the valuing stage, where individual values are organized as part of a person's value system, these individual values are not likely able to explain for the person's subsequent decisions and actions. Rather, it is a person's entire value system which guides his/her responses to different situations. Second-hand values (Tripp, 1993) and competing values often come into play in a person's process of responding to different life situations. This relates to Hill's (1991) emphasis to consider the volitional aspect differently from the cognitive and affective aspects when examining values. Thus, observed inconsistencies between beliefs and actions (Sosniak, Ethington, & Varelas, 1991; Thompson, 1992; Tirta Gondoseputro, 1999) may be explained from a perspective of personal value systems, taking into consideration the interaction among competing values and the influence of second-hand values.

Values in the mathematics classroom

In view of the common misconception of the nature of values in the mathematics classroom (which was discussed earlier in this paper), this second theoretical framework will be explained in greater detail.

In exploring these values, Bishop (1996) has identified three categories of values of interest --- general educational, mathematical, and specifically mathematics educational: For example, when a teacher admonishes a child for cheating in a test, the values of 'honesty' and 'good behaviour' derive from the **general educational** [emphasis added] and socialising demands of society. Then when a teacher proposes and discusses a task such as the following: "Describe and compare three different proofs of the Pythagorean theorem" the **mathematical** [emphasis added] values of 'rationalism' and 'openness' are being conveyed. However there are other values being transmitted which are specifically associated with the norms of the institutions within which **mathematics education** [emphasis added] is formally conducted. For example, consider the following instructions from the teacher: "Make sure you show all your working in your answers", "Don't just rely on your calculator when doing calculations, try estimating, and then checking your answers", the values implied are all about 'examination-wisdom' and 'efficient mathematical behaviour'. (Bishop, 1998, p. 34)

Mathematical values.

In particular, Bishop (1988) suggests that mathematical values may be considered in terms of three pairs of complementary values, i.e. rationalism and objectism, control and progress, openness and mystery. Each pair corresponds to White's (1959) three components of culture --- ideology, sentiment, sociology --- respectively.

Rationalism involves the separation of an idea from any associated 'object', and focuses on utilizing "deductive reasoning as the only true way of achieving explanations and conclusions" (Bishop, 1988, p.62). Complementary to this is the value of **objectism**, which illustrates the power of mathematics as one which deals with abstract ideas efficiently by concretizing them, treating these ideas as if they are objects. Both these two values, "more than any other qualities, are what have enabled Western mathematics to 'conquer' the world of mathematics, and to be valued above others" (Bishop, 1991, p. 202).

In the sentimental dimension, the value of **control** reflects the feeling of security offered by the subject over not just natural phenomenon, but also when mathematics is applied to solve problems in the social environment. Whereas this value has the flavor of stability, mathematics is at the same time about change and **progress**.

Mathematics exemplifies the value of **openness**, through which the subject achieves transparency in its ideas and conclusions, and arguments are critically analyzed and discussed. Mathematical truths are expected to pass the test of critical examinations and analysis, and in many ways this has been extended to other truths and theories as well. Another aspect of openness, however, is that instead of establishing the validity of arguments and propositions, weaknesses, deficiencies and errors are exposed of other such arguments and ideas.

In any case, mathematics as a subject has not lost its **mystery**. Even mathematicians are lured to the subject and attracted to the challenge it offers in virtue to the mystery it represents. Its link to computer programming and technology, coupled with the vast advances in information technology, has only made mathematics more mysterious.

That these mathematical values are complementary pairs suggest that balanced pupil exposure to all of them is necessary for the internalization of a positive outlook towards mathematics. However, are the complementary pairs portrayed with equal emphasis in the real-life mathematics classroom? In particular, for this study, what is the contribution towards this by the mathematics textbooks?

Mathematics educational values.

The professional teaching practice of mathematics teachers and the role of textbooks as 'invisible teachers' portray values which are mathematics educational in nature. Although 'Western' mathematics may be taught in different countries (and so we expect the same mathematical values to be represented and transmitted in different places), socio-cultural, pedagogical and individual factors inherent among different classrooms and different school systems lead to differences in agreement as to what constitute desirable values (e.g. freedom in choosing problem-solving strategies) within the context of mathematics education.

Amongst the large number of mathematics educational values, this study focuses on five complementary pairs. Their being chosen reflects what we feel are the more recurrent examples of mathematics educational values which are represented and transmitted in mathematics classrooms today.

In the formalistic-activist view continuum (Dormolen, 1986), the **formalistic** end sees mathematics learning as involving deductive reasoning and receptive learning. Mathematics is considered as a cultural heritage worth preserving. The **activist**, however, views mathematics learning as involving intuitive reasoning and discovery learning. Mathematics is thus a human activity which is very much alive and relevant. A comparable model may be Cobb's (1988) imposition/transmission --- negotiation/active construction continuum.

The next continuum to be examined is derived from Skemp's (1979) notion of **instrumental** vs **relational** understanding/learning. The former is related to learning rules, procedures and formulae; the latter emphasizes pupil relating a task to appropriate schema.

The two pairs of values discussed above are derived from pedagogical considerations. Culturally, how is mathematics perceived? Here, three pairs of values can be broadly conceived, answering the questions of what, who and why.

What is the nature of mathematics learning and teaching? The **relevance** of mathematical knowledge and skills allows us to solve daily problems and create more ingenious ways through which the society progresses. Foreign mathematical systems may be different from the 'Western' model because mathematics provide solutions to specific and particular cultural needs and demands. On the other hand, mathematics may be taught and learnt as **theoretical knowledge**, often without any familiar everyday context.

Who can be involved in mathematical activities? In the **accessibility** --- **specialism** continuum, we can identify with mathematics teaching which either advocates mathematics for all or mathematics for the selected, elite group of gifted mathematicians.

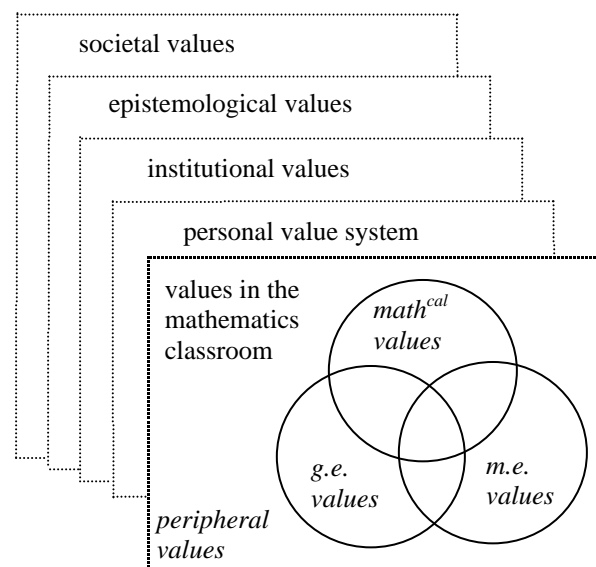
Why is mathematics taught and learnt? The performance expectations listed in Robitaille (1993) range from knowing, using routine procedure, investigating and problem-solving, reasoning, to communicating. Within the parameter of applying mathematical knowledge, these can be broadly conceptualized as being located at two ends of an **evaluating** --- **reasoning** continuum. The first three performance expectations deal with using mathematical knowledge to evaluate unknown answers; the last two, on the other hand, have to do more with using mathematical knowledge to supplement our capability to reason and to communicate ideas.

Mathematical and mathematics educational complementary value pairs may not be exhibited at any one decision point. For example, there are situations when the teacher or the textbook recognizes that the best pedagogical approach is to teach particular algorithms for pupil substitution of known values. Are we to conclude from this incident *alone* that the teacher embraces instrumental understanding per se? Perhaps this provides a third explanation for observed inconsistencies between actions, and beliefs and attitudes (as discussed earlier)?

Since culture and values are closely related, the nature of any given value in the mathematics classroom is relative to the socio-cultural setting. How are these values related to one another, and to other values in the wider context? Figure 1 illustrates a proposal for the relationship existing between and amongst values.

In Figure 1, general educational, mathematical and specifically mathematics educational values do not exist mutually exclusive of one another. After all, some values fit into two or all three of the categories. For example, progress and its associated value of creativity is as much a mathematical and mathematics educational value as a general

Figure 1
Relationship of values



Note. math^{cal} values --- mathematical values
 m.e. values --- mathematics educational values
 g.e. values --- general educational values

educational value. Together, these values are portrayed through the operating functions of teachers, textbooks, syllabi, etc. These are usually accompanied by a range of (more implicit) peripheral values, which include the implicit message portrayed by teacher dressing (Neuman, 1997, Chap. 14), textbooks' physical designs, for examples.

Beyond the mathematics classroom, values are situated in increasingly larger contexts of personal, institutional, epistemological, and societal values. It may be recognized that the layers in this tentative model reflect the influence of Billett's (1998) five levels of knowledge genesis. Eckermann (1994, Figure 2.1) has also attributed similar factors to influences on the nature of teachers' teaching philosophies. Note the set of epistemological values as are continually being constructed and modified by mathematicians and mathematics education scholars. This highlights another aspect of human contribution to the nature of mathematics in any cultural context. Similar categories have been discussed with reference to science education variously as 'epistemological and supporting values' (Tan, 1997) and 'values of science and research ethics' (Allchin, 1999).

The 'soft' boundaries separating the different value categories challenge the notion of interaction among values as 'top-down'; rather, there is a sense of two-way permeability. For example, institutional values shape values in the mathematics classroom as much as values in the mathematics classroom influence the development of institutional values.

Neither is any value category situated wholly within a 'larger' set of values. Values which do not lie within the boundaries of the larger contextual value categories represent 'minority' values arising from an increasingly multicultural classroom and societal contexts.

Mathematics textbooks as source and medium of values portrayal.

The textbook writer, editor, publisher, together with their country's political and educational authorities, determine what gets published, how the content is portrayed, the tone adopted, etc. Like mathematical knowledge itself, they are all value-laden in their own ways. Thus, textbooks have been shown to propagate officially sanctioned worldviews of political and educational bodies (Anyon, 1983; Apple & Christian-Smith, 1991; Brummelen, 1991).

In the area of mathematics, Fauvel's (1991) analysis of three 'old' mathematics textbooks published between the mid 1500s and early 1900s reveals that values were consciously and actively built into the text through the skilful incorporation of such techniques as teaching through dialogue, adopting a didactic catechism style, and reflecting on the links between historical past and current methods and instruments.

That present-day textbooks adopt a more direct exposition teaching approach and incorporate a greater number of drills and practice questions (McGinty et al, 1986) do not imply that mathematics textbooks have become value-free. Fauvel (1991) and Shield (1998) show how non-subject specific messages are still being presented to readers. McBride's (1994) examination of two American mathematics textbooks demonstrates the representation of political values, and argues that reader-environment issues transmitted in these books are out-of-focus. After all, "whenever an utterance is made, the speaker/writer makes choices (not necessarily consciously) between alternative structures and contents" (p.3). Dowling's (1991) analysis of the British SMP 11-16 textbook series demonstrates that the different books intended for different pupil ability actually transmit strong values along the academic/mundane and intellectual/manual lines, and thus act as a sort of social class gatekeeper. Dowling's (1996) analysis of the similar textbook series also notes that values are transmitted through the recruitment of different reader voices.

Values representation through textbooks may perhaps be part of the entire *education* process after all. As Brummelen (1991) notes

while it is understandable that authors do not wish to present children with unduly depressing views of life, burying or glossing over basic problems and key value conflicts in society is untruthful and fails to help students even sense the need for social transformation, let alone prepare them for social and economic empowerment. (p.216)

As "teachers at a distance" (Fauvel, 1991, p. 116), textbooks often define the facts to be learnt within the school's curriculum (Dorfler & McLone, 1986). They set "the style and order in which the material is covered in the course" (Mestre, 1988, p. 204).

"Teachers rely on them to organise lessons and structure subject matter" (Tyson-Bernstein, 1988, p. viii). Although teachers in Beijing, Hong Kong and London in Leung's (1992) study did not perceive textbooks as highly influential on their own teaching practices, his data conclude that nearly "all the lessons in the three places were very much influenced by the textbooks in use" (p. 197). Likewise, textbook usage in Singapore and Victorian (Lokan, Ford, & Greenwood, 1996) secondary mathematics classrooms is high.

Values in mathematics education: An operational definition

Values in mathematics education, then, represent one's internalization and 'cognitization' of affective variables (such as beliefs and attitudes) related to the discipline of mathematics, to one's community, and to the wider world through one's experience with the learning or teaching of mathematics. These values form part of one's ongoing developing personal value system, which equips one with a pair of cognitive and affective lenses to shape and modify one's way of perceiving and interpreting mathematics and the world, and to guide one's choice of course of action.

It is not any individual value --- but rather, one's value system --- which influences one's decisions and actions in life. Even then, this influence is in the form of guidance, not control. This can be understood in terms of competing values, second-hand values and the nature of some values to exist in complementary pairs. Thus, although personally-held values are deeply internalized, one's value system is not a static entity. They are constantly being challenged and modified as part of one's life experience.

Research questions

In the light of the discussion thus far, this preliminary analysis of text in representative Years 7 and 8 (the first two years of secondary schooling) mathematics textbooks in Singapore and Victoria attempts to answer the following research questions:

- (a) How are the mathematical and mathematics educational values emphasized relative to their complementary values in Singapore and Victoria textbooks?
- (b) What accounts for any difference in the ways mathematical and mathematics educational values are portrayed in Singapore and Victoria textbooks?

A short discussion of the consequences and implications of such differences follows.

Methodology

The analysis of textbooks belongs to a class of nonreactive research technique called content analysis. This technique is useful for this study as it "can reveal messages in a text that are difficult to see with casual observation" (Neuman, 1997, p. 274).

The first two years of secondary education in Singapore (Secondary 1 and 2) and Victoria (Years 7 and 8) are selected to help define the population and sampling frame. These years also correspond to the early impressionable teenage years. Furthermore, the Secondary 3 and 4 mathematics syllabi in Singapore is produced in collaboration with the University of Cambridge in Britain; thus, the mathematics curriculum in those higher grade levels may not totally reflect Singapore's own intentions.

The extent to which particular textbooks are representative was informed by random surveys of government, Catholic and independent schools in Singapore and Victoria. The two series adopted by the most number of schools in each region were identified for this study. Over two grade levels, then, a total of eight books were selected for analysis, coded as A1-7, A1-8, A2-7, A2-8, S1-7, S1-8, S2-7, and S2-8.

Without the luxury of time, only certain topics of each textbook were analyzed. The 112 chapters in the 8 textbooks were grouped into 24 topics according to strands of algebra, chance and data, mathematical tools, measurement, number, and space. From these 24 topics, the 2 most emphasized topics (by percentage space) were identified, namely, 'rate, ratio and percentage' (number) and 'area, perimeter, volume' (measurement), accounting for a total of 680 pages, and ranging from 15.88% (S1) to 23.36% (A1) among the textbook series.

Only text was analyzed, which includes expository writing, worked examples, practice questions, and peripheral writing (introductory remarks, meta-expositions, summaries, etc). Cover designs, illustrations, figures, charts and physical properties of text (e.g. font type, font size, text length) were excluded in this preliminary study. Latent coding (Neuman, 1997) was used to analyze the often implicit meanings in text content at various levels ranging from the lexical to the contextual. While validity of such semantic analysis is very high (Neuman, 1997), reliability needs to be taken care of. This was achieved through practice rounds with other text in the same textbooks, construction and use of written rules, and piloting these rules with peers.

A checklist was compiled to facilitate the identification and coding of text features which act as value signals. Some of these value signals had been acknowledged before (Bishop, 1988; Dowling, 1996), whereas the rest were constructed through random trial coding with other text in the same textbooks. To ensure consistency, new specific coding situations which arose after coding began were all recorded, followed by repeated perusals of text which had been analyzed.

The assumption here is that a mathematical or mathematics educational value may be portrayed by one or more of the value signals. On the other hand, a value signal may communicate more than one value. Another assumption made was that the strengths of the messages portrayed by the value signals are mutually similar. Thus, the effect of each value signal is not weighted. The extensive range of value signals representing particular values may, however, neutralize any effect (if any) somewhat arising from this assumption.

Symbolization of ideas and quantities using numbers, letters, symbols, equations and inequalities is heavily utilized. A limitation of this study is that counts of symbolization were estimated from ten randomly chosen pages among the chosen topics in the four textbook series. A precise count is not only not feasible given the time constraint, but it would distract one from the identification of other value signals in the text. As it turned out, the use of symbolization is so high (see Table 1 in the next section) that either the estimated or actual number would emphasize the value of objectism anyway.

Results

19 856 value signals were identified from the 680 pages analyzed, 12 433 of which are featured within the 444 pages of Victoria textbooks. Furthermore, 28 937 counts of symbolization were estimated to have been featured among the chapters analyzed.

Portrayal of mathematical values

The number of value signals for each mathematical value, and the relative proportions between complementary pairs, coded in Singapore's and Victoria's textbook series are shown in Table 1:

Table 1
Distribution of mathematical value signals

	Rationalism	Objectism
A1	486 (0.05 [†])	41 + 8 602 [†] (0.95 [†])
A2	384 (0.05 [†])	44 + 7 682 [†] (0.95 [†])
S1	340 (0.05 [†])	49 + 6 083 [†] (0.95 [†])
S2	464 (0.07 [†])	63 + 6 570 [†] (0.93 [†])
A1 + A2	870 (0.05 [†])	85 + 16 284 [†] (0.95 [†])
S1 + S2	804 (0.06 [†])	112 + 12 653 [†] (0.94 [†])
All	1 674 (0.05 [†])	197 + 28 937 (0.95 [†])
<hr/>		
	Control	Progress
A1	980 (0.97)	30 (0.03)
A2	1 212 (0.98)	30 (0.02)
S1	476 (0.98)	9 (0.02)
S2	561 (0.99)	4 (0.01)
A1 + A2	2 192 (0.97)	60 (0.03)
S1 + S2	1 037 (0.99)	13 (0.01)
All	3 229 (0.98)	73 (0.02)
<hr/>		
	Openness	Mystery
A1	469 (0.16)	2 431 (0.84)
A2	403 (0.13)	2 749 (0.87)
S1	193 (0.10)	1 744 (0.90)
S2	205 (0.08)	2 214 (0.92)
A1 + A2	872 (0.14)	5 180 (0.86)
S1 + S2	398 (0.09)	3 958 (0.91)
All	1 270 (0.12)	9 138 (0.88)

Note. [†] estimated frequencies of symbolization.

In general, then, both Singapore and Victoria mathematics textbooks emphasize the mathematical values of objectism, control and mystery.

The capability and flexibility with which mathematics manages the abstract through concretizing it with symbols is clearly portrayed in both regions. On the other hand, the message of rationalism --- of separating object from idea --- is mainly achieved through the absence of context (e.g. Case 1 below). In Singapore textbooks, rationalism is also strongly communicated through the use of logic connectors such as 'thus', 'so', 'therefore' and 'hence', all of which imply cause-and-effect and deductive reasoning.

Case 1
Absence/removal of context (A1-7-7-227P)

The length of one pair of opposite sides of a square is reduced by 10% and the other pair of sides is increased by 10%. What is the area of the new rectangle compared with the original square? (Is it the same, a certain percentage greater, or smaller?)

The value of control is presented mainly through the use of imperatives. For example, in Case 2, the reader is to

Case 2

Use of imperatives and portraying personal empowerment (S1-7-8-222Q)

Calculate in each shop the difference between the hire purchase price and the cash price. State which shop is offering the better deal.

Incidentally, the portrayal of personal empowerment through mathematics as is illustrated in Case 2 is also substantially used by textbooks in both regions to communicate the value of control.

Worked solutions in textbook series A2 are also accompanied by step-by-step instructions which spell out clearly what is to be carried out:

Case 3

Use of imperatives and detailed instructions (A2-8-4-155E)

Worked Example 4.7

Express the first amount as a percentage of the second in each case

(a) 42, 70

Steps

- (a) 1. Write a fraction with the first amount as the numerator and the second as the denominator.
2. Multiply the fraction by $\frac{100}{1}$ and add a % sign.
3. Cancel down. Here we have divided
4. Multiply.

Solution

....

Although mathematics is not highly conveyed as facilitating progress, this value is signaled differently in the two regions. In Singapore, progress is conceptualized somewhat more theoretically as in the generation of new mathematical knowledge, such as in using the formula for the area of rectangles to derive the formulae of areas of triangles, parallelograms and trapeziums (S1-7-12-333P, S2-7-9-162E). In Victoria, there is a promotion of mathematical knowledge contributing to societal progress, like the anthropologist's estimation of human or animal heights from excavated skeleton parts (A1-8-8-294P), the use of scale drawings in the precise descriptions in maps and building plans (A2-8-10-451E), and the use of pantographs in the pre-photocopier era (A1-8-7-241P).

The pre-eminence of the value of mathematics being mysterious over that of mathematics being open to scrutiny is attributed in both Singapore and Victoria to the use of imperatives and of specialist vocabulary. The use of the passive voice (thus removing the subject in the sentence) is a third major factor in Singapore text, although this is not as significantly used in Victoria textbooks. Imperatives instruct the reader and suggest 'one best way' to do mathematics. Specialist vocabulary classifies readers into in- and out-groups. Case 4 illustrates:

Case 4

Use of imperatives and specialist vocabulary (A1-7-9-300P)

Then draw a graph as shown: plotting length and area. Join the points to make a curve. From the graph, find which length gives the greatest value for area.

The most frequently adopted value signal for openness in Victoria textbooks is the use of pronouns ‘we’ and ‘you’, and their related forms (e.g. ‘us’, ‘your’). The use of ‘you’ and its related form in these books --- often to refer to the operation of some menial tasks --- reminds that ‘we’ refers to the textbooks writers only. In other words, openness is conveyed through a sense of second person participation, but not through the establishment of peer relationship between writer and reader (e.g. Case 5).

Case 5
Use of ‘we’ and ‘you’ (A2-8-4-147E)

So, to convert 95% to a decimal, we may simply divide by 100. You may recall from decimals in Year 7 that to divide by 100, we place a decimal point after the units digit.

In Singapore textbooks, however, openness is most often conveyed through inviting reader question posing, a value signal which was not employed in any of the Victoria text analyzed in this study (e.g.: Case 6).

Case 6
Inviting reader question posing (S2-7-11-211P)

- Understand the problem by asking the questions:*
- 1. How far can a car travel on 1 litre of petrol?*
 - 2. How much petrol is needed to travel 1 km?*
 - 3. How many litres of petrol are required to travel 260 km?*

Portrayal of mathematics educational values

Table 2 shows the distribution of mathematics educational value signals in the Singapore and Victoria textbooks analyzed.

Table 2
Distribution of mathematics educational values

	Formalistic view	Activist view
A1	3 330 (0.93)	237 (0.07)
A2	3 398 (0.94)	229 (0.06)
A1 + A2	6 728 (0.94)	466 (0.06)
S1	1 618 (0.89)	207 (0.11)
S2	2 193 (0.91)	225 (0.09)
S1 + S2	3 811 (0.90)	432 (0.10)
All	10 539 (0.92)	898 (0.08)
	Relational understanding	Instrumental understanding
A1	39 (0.02)	2 010 (0.98)
A2	55 (0.03)	2 119 (0.97)
A1 + A2	94 (0.02)	4 129 (0.98)
S1	23 (0.04)	523 (0.96)
S2	25 (0.03)	789 (0.97)
S1 + S2	48 (0.04)	1 312 (0.96)
All	142 (0.03)	5 441 (0.97)
	Relevance	Theoretical
A1	230 (0.41)	330 (0.59)

A2	194 (0.38)	315 (0.62)
A1 + A2	424 (0.40)	645 (0.60)
S1	85 (0.35)	159 (0.65)
S2	116 (0.37)	195 (0.63)
S1 + S2	201 (0.36)	354 (0.64)
All	625 (0.38)	999 (0.62)

	Accessibility	Specialism
A1	57 (0.05)	1 206 (0.95)
A2	40 (0.03)	1 216 (0.97)
A1 + A2	97 (0.04)	2 422 (0.96)
S1	15 (0.01)	1 032 (0.99)
S2	19 (0.01)	1 302 (0.99)
S1 + S2	34 (0.01)	2 334 (0.99)
All	131 (0.03)	4 756 (0.97)
	Evaluation	Reasoning
A1	2 062 (0.90)	223 (0.10)
A2	2 157 (0.93)	154 (0.07)
A1 + A2	4 219 (0.92)	377 (0.08)
S1	630 (0.93)	50 (0.07)
S2	830 (0.97)	22 (0.03)
S1 + S2	1 460 (0.95)	72 (0.05)
All	5 679 (0.93)	449 (0.07)

The text in both Singapore and Victoria has been found to convey the mathematics educational values of formalistic view, instrumental understanding, theoretical knowledge, specialism, and evaluating with greater emphases than their respective complementary values.

Through the predominant use of specialist vocabulary and inclusion of routine drills questions in textbooks, the two regions emphasize the formalistic view of mathematics. Singapore is likely to use more specialist vocabulary than to include individual drills questions, whereas the reverse is true in Victoria. Case 7 is an example communicating implicitly the messages of both deductive reasoning and receptive learning.

Case 7

Use of specialist vocabulary in a drills question (A2-8-5-198Q)

Use the formula $C = \pi D$ to calculate the circumference of the following circles.

Practice questions which appear to be of the investigative/problem-solving nature in Singapore textbooks are likely to have been introduced in the worked examples given earlier in the respective sections. Problem-solving strategies are also explicitly taught in the worked examples. These presentation styles reinforce the formalistic view of mathematics learning in Singapore.

The remaining truly investigative/problem-solving type of practice questions in Singapore textbooks do portray an activist component to mathematics learning. This is the predominant value signal in Victoria text too. In addition, the Victoria textbooks are more likely to portray this value through relating mathematical knowledge to its historical development, thus exposing the range of intuition, trails and errors, and estimation experienced by mathematicians. Case 8 is an example.

Case 8

Relating current mathematical knowledge to historical development (A1-8-8-270E)

*Archimedes, about 250BC, stated the following: 'The area of a circle is the same as the area of a triangle which has a base equal to the circumference of the circle and an altitude equal to the radius of the circle'. This is how he argued it!
Divided [sic] the circle into concentric rings.
Cut the circle along a radius and open out the rings into straight sections and you have a step triangle.
If you make the rings as thin as possible, it will become exactly equal to the triangle!
... ..*

Among the text analyzed, questions testing for reader mathematical reasoning are only found in the Victoria textbooks. The activities involved in such questions (e.g. generalizing, justifying) also exemplify the activist view (e.g. Case 9).

Case 9

Performance expectation --- mathematical reasoning (A1-8-6-198Q)

*Tennis balls can be bought in a can of 3 for \$8.88 or a can of 4 for \$10.64.
(a) Find the cost of 1 ball in each case. Which is cheaper?
(b) Consider that you are only going to play one afternoon of tennis on one court. Which can will you buy?*

Both Singapore and Victoria promote mathematics learning as involving instrumental understanding. This message is most frequently conveyed through the inclusion of a large number of drills questions (as opposed to problem-solving sums). Interestingly, there is a larger number of drills questions (by percentage space) in Victoria textbooks than in Singapore textbooks. Textbook S2 even 'prepares' the student reader for different possible problem scenarios by including relevant worked examples, which also promotes the formalistic view.

The relevance of mathematics learning is mainly signaled through the inclusion of local context, as well as through demonstrating human control over his/her environment. In Case 10, the value of relevance is especially obvious in the summer season!

Case 10

Power of mathematics to control in a local context (A1-8-8-273Q)

A fire spotter can see from his tower a distance of 53km in all directions. How many sq. kilometres of forest can he supervise?

Local context in Victoria textbooks are largely achieved through the use of authentic, local documents or data; in Singapore, local context comes through most often as a result of labeling people and objectives with local names (e.g. Cases 11 and 12).

Case 11

Use of local context in Victoria textbook (A1-8-6-179Q)

The numbers of telephone services in operation in Australia were as follows:

<i>Year</i>	<i>1985</i>	<i>1986</i>	<i>1987</i>
<i>Services in operation</i>	<i>6 186 835</i>	<i>6 501 468</i>	<i>6 816301</i>

Find the percentage increases from 1985 to 1986 and from 1986 to 1987. (Express each answer correct to the second decimal point.)

Case 12

Use of local context in Singapore textbook (S2-8-1-11/12E)

EXAMPLE 10

In 1995, Mr Tan received a 10% pay rise. The following year his pay was cut by the same percentage when his company turned in a poor performance. After the pay cut, did Mr Tan take home more or less than before the pay increase?

... ..

Strategy 1:

Suppose that Mr Tan's original pay was made up of 10 parts.

Strategy 2:

Suppose Mr Tan's pay was \$100 before the pay increase.

Strategy 3:

Let Mr Tan's pay before the pay increase in 1995 be \$x.

... ..

Although both Singapore and Victoria are multicultural societies, it is only in Victoria textbooks that readers are likely to be invited to contribute their own cultural experiences/knowledge, another value signal for relevance:

Case 13

Inviting readers to contribute cultural experience/knowledge (A2-7-7-307P)

We have given the history of the calendar we use today. Find out something about the calendar used by the Mayas of Central America and the Chinese calendar.

The theoretical nature of mathematics education is often achieved through the removal of context. There are also instances in both Singapore and Victoria when impractical context are introduced. Case 14 is an example of the latter:

Case 14

Impractical context (i.e. subscription fee is not usually determined this way) (S1-7-7-182Q)

The subscriptions to a club for men and women are in the ratio 4:3. Two men and five women pay a total sum of \$460. What is the subscription fee for each man?

A sense of specialism of mathematics learning is also emphasized in Singapore and Victoria textbooks, mainly conveyed through the use of specialist vocabulary. For example, in Case 15, the definition of a right prism may not be easily understood without

possibly referring to other text. Does this imply that readers who are already familiar with the specialist vocabulary used are further privileged to acquire another new mathematical concept faster or sooner?

Case 15

Use of specialist vocabulary (S2-7-10-182P)

In general, a right prism is a solid which has two parallel ends of the same shape and size. Also, its lateral surfaces are perpendicular to its parallel ends.

In the last value category, the practice questions in the Singapore and Victoria text analyzed have been found to test predominantly for reader performance on knowledge, using routine procedures, and investigations/problem-solving. There are little opportunities for readers to use their mathematical knowledge to reason and to communicate ideas and opinions (e.g. Case 9). As in Case 16 below, potential exists for practice questions to be phrased differently so as to stimulate more context-specific reasoning exercises:

Case 16

A drills question (S2-8-1-7Q)

The rice consumption in a certain city was 80 000 tonnes in 1985. By 1990, the rice consumption had increased by 24%. If the consumption of rice in the city continued to increase at the same rate, i.e., 24% every 5 years, find the consumption of rice in the city in 1995.

Discussion

In the Singapore and Victoria textbooks analyzed, there is a predominant emphasis of the mathematical values of

objectism	over	rationalism,
control	over	progress,
mystery	over	openness,

and the mathematics educational values of:

formalistic view	over	activist view of mathematics learning,
instrumental understanding	over	relational understanding in mathematics learning,
theoretical	over	relevant nature of mathematical knowledge,
specialism	over	accessibility of mathematics learning, and
evaluating	over	reasoning aspect in mathematics learning.

The way mathematical and mathematics educational values are portrayed in Singapore and Victoria textbooks, school mathematics involves the receptive learning of concepts, formulae, structure and theorems (formalistic view of mathematics learning), the schemas of which are fixed and stable (control). The mathematical situation is more often theoretical, and the learner is expected to demonstrate the ability to concretize and objectivize any abstract idea (objectism), so as to match the given task to particular and appropriate formulae or theorems (instrumental understanding). Thus, school mathematics is learnt to help us find numerical answers to unknowns (the evaluating aspect of mathematics learning). Nevertheless, the nature of mathematics remains a mystery to all (mystery), and as such the knowledge is not readily accessible to everyone (specialism).

What are the implications of such a portrayal of mathematical knowledge and mathematics learning on pupils? By the very nature of content analysis, the impact and effects of these subtle messages on pupil readers cannot be inferred without carrying out a further study on their perceptions (Neuman, 1997; Sommer & Sommer, 1997). However, the potential remains for young pupil readers to be influenced by the way textbooks perceive mathematics and mathematics learning. If such a relation between textbook portrayal and reader perception exists, the effects on pupil attainment of cognitive objectives can be very real. For example, mathematics is portrayed in these textbooks as involving more of concretizing and objectivizing abstract ideas than abstracting ideas for rational and reasonal processing. Yet we are more familiar with the latter operation in our lives (Bishop, 1988). Hence we hear pupils complain that mathematics is 'too abstract to understand'. From a wider perspective, the implications for a nation like Singapore whose citizens is her only natural resource can be disturbing indeed. In fact, this would possibly jeopardize Singapore's realization of the current 'Thinking Schools, Learning Nation' initiative in the knowledge-based economy of today.

The more commonly adopted value signals for mathematical values --- i.e. symbolization, imperatives, specialist vocabulary, and the passive voice --- have somehow become an accepted feature of the nature of mathematical writing. These value signals also enjoy a relative ease of deployment and of being repeatedly used in mathematical text.

As for mathematics educational values, the fact that they are attributed to teaching practice suggests that their portrayal are mostly within the control of textbook writers. An exception may be the amount of specialist vocabulary used. Specialist vocabulary communicates the values of mathematics being mysterious as well as of mathematics learning as formalistic and open to certain people only. Although there have been efforts made by textbook writers and teachers alike to reduce technical jargon in their text/teaching, there is often a limit to which this can be maintained without sounding too general and vague. This situation is certainly true of mathematics topics at the secondary level.

However, the form of genre specific to any discipline is continually evolving and developing as a response to the environment to which it is applied. There is certainly an element of human participation and intervention in this process. Thus, while the realization of a balanced emphasis on affective values in mathematics education may rest on the prevailing nature of mathematical writing, opportunities exist for value signals to be deployed in different ways so as to bring about this desired portrayal of relevant mathematical and mathematics educational values.

Victoria's greater likelihood to integrate (rather than include) local context, to relate mathematical situations to their respective historical developments, and to invite readers to contribute cultural mathematical knowledge, provide the Victoria textbook reader with more opportunities to encounter mathematical situations involving subjective or debatable interpretations and different possible solutions. For example, different pupils may justify their different answers to a question involving an authentic, local train time-table, citing real-world or personal experiences. Even though problem posing provides opportunities for a more open-ended learning experience (Silver, 1994), the way it is introduced (uniquely) in the Singapore text analyzed (e.g. Case 6) suggests its use as a focussed problem-solving strategy only. In fact, the variety of such strategies are explicitly taught

in the Singapore textbooks analyzed. The inclusion of worked examples to teach question posing is unique to Singapore textbooks only. Opportunities for different interpretations and possible solutions in mathematics are also extended to practice questions in Victoria; Singapore textbooks do not feature questions testing for mathematical reasoning and communication. This is despite an aim of the Singapore mathematics education being for pupils to “use mathematics as a means of communication” (Singapore Ministry of Education, 1990, p. 2). On the other hand, there is an inclusion of a mathematical project component in Victoria’s tertiary entrance examinations.

Comparing practice questions alone, it is thus not surprising to find a majority of questions in Singapore and Victoria textbooks testing for pupil knowledge and drills. However, this proportion is smaller among Singapore practice questions than Victoria practice questions. Possible reasons are Singapore’s more explicit official emphasis on problem-solving (Singapore Ministry of Education, 1990, p. 3), and the relative ease and affordability with which supplementary question assessment material is available in bookshops around Singapore.

Singapore textbooks’ apparent cautiousness against assessment involving subjective or multiple solutions may reflect the writers’ internalization of some time-tested, more traditional mathematics educational values, to the extent that even problem-solving questions appear in the form of drills! This may well explain why Australia traditionally outperforms Singapore in International Mathematical Olympiads, but Singapore’s lower secondary pupils top the Third International Mathematics and Science Studies.

Education remains the key to national survival and individual social mobility in many countries. Intellectual capital is also increasingly a defining factor for a nation’s competitiveness. The nurturing of the different domains of educational objectives and learning more about how they work with one another provide a perspective towards the creation of knowledge in the 21st century. This preliminary study, hopefully, makes a contribution in this regard.

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